Ozsvath

cube of resolutions for knot Floer homology

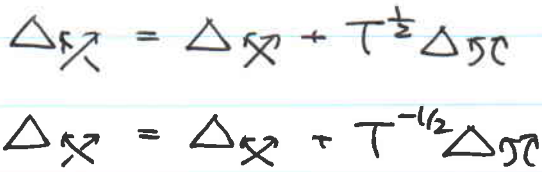
braid presentation

joint work with ① A. Stipsicz & Szabó — A

② Szabó — B, C

Steps

A. Define knot Floer homology for singular links 

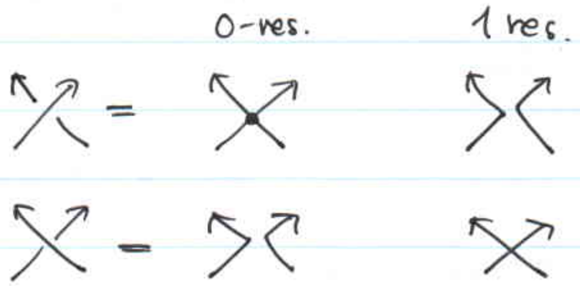
B. 

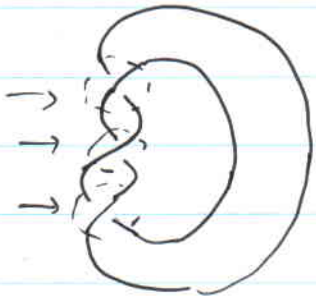
Skein relation for Alexander poly

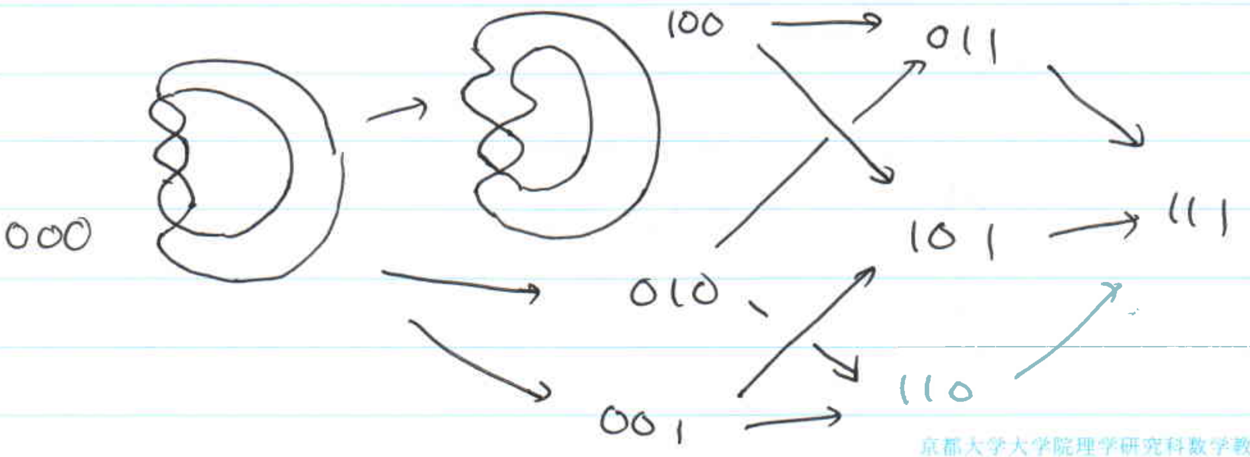
⇝ Find long exact sequence

C. Calculate HFK for planar singular knot in braid form

Draw knot in braid form

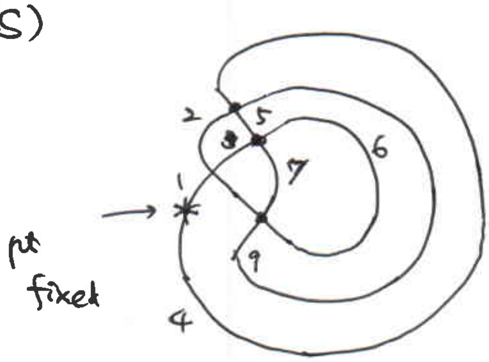






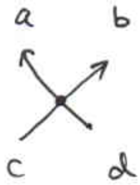
S = singular knot in braid form

$\mathcal{A}(S)$



$\mathbb{Z}[u_1, \cdots u_n, t^{-1}, t]]$

↑ corr. to edges ↗

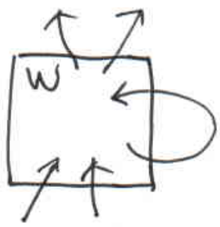


$t^2 u_a u_b = u_c u_d$

$t u_a + t u_b = u_c + u_d$

new rel

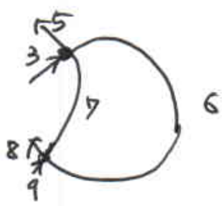
W collection of vertices



$t^{|W|} \prod u_{out} = \prod u_{in}$

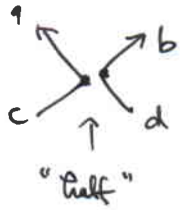
$|W| = 2 \#$ doublepts in W

In above example



$t^4 u_5 u_8 = u_3 u_9$

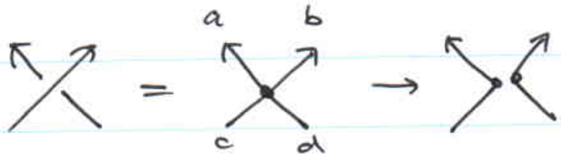
(



$t u_a = u_c$

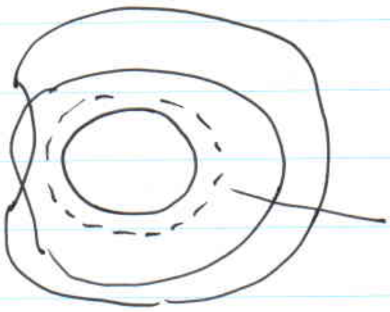
$t u_b = u_d$

dim alg. = Euler char. of singular knot

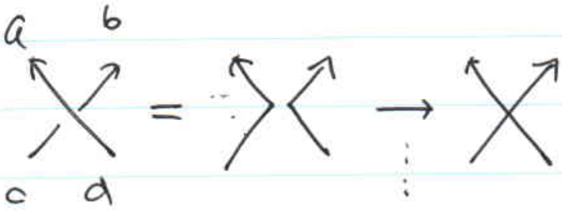


$t^2 u_a u_b = u_c u_d \Longleftarrow \begin{array}{l} t u_a = t u_c \\ t u_b = u_d \end{array}$

NB.



$t^{|W|} = 1 \Rightarrow$ ring is trivial.



mult. by $t u_a - u_d$

$\mathcal{A}(S)$ is bigraded

u_i drops Maslov gr. by 2

drop Alex. by 1

<u>Thm</u> (O-Szabó)

$H_*(C, D) \cong HFK^-(K) \otimes \mathbb{Z}[t^{-1}, t]]$

— over $\mathbb{Z}[t^{-1}, t]]$

— # of generators smaller than grid diagrams

$$\left(\Sigma_g, \{\alpha_1 \dots \alpha_g\}, \{\beta_1, \dots, \beta_g\}, \overset{w}{\mathbb{O}}, \overset{z}{\mathbb{X}}\right)$$

$$\rightsquigarrow \Sigma_g, \quad \begin{matrix}\{\alpha_1, \dots, \alpha_{g+n-1}\} \\ \{\beta_1, \dots, \beta_{g+n-1}\}\end{matrix}, \quad \left\{\begin{matrix} O_1 & \dots & O_n \\ X_1 & \dots & X_n\end{matrix}\right\}$$

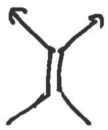
$$\Sigma_g - \alpha_1 \dots - \alpha_{g+n-1} = A_1, \dots, A_n$$

each A_i has one X_{m_i} one O_i

$$\Sigma_g - \beta_1 \dots - \beta_{g+n-1} = B_1 \dots B_n$$

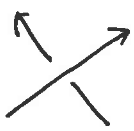
B_i has one X_{m_i}, O_{m_i}

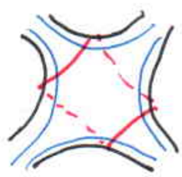
singular knot

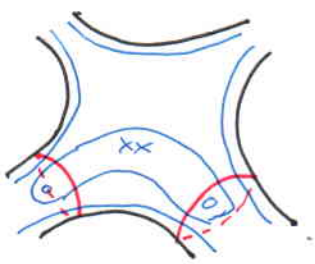


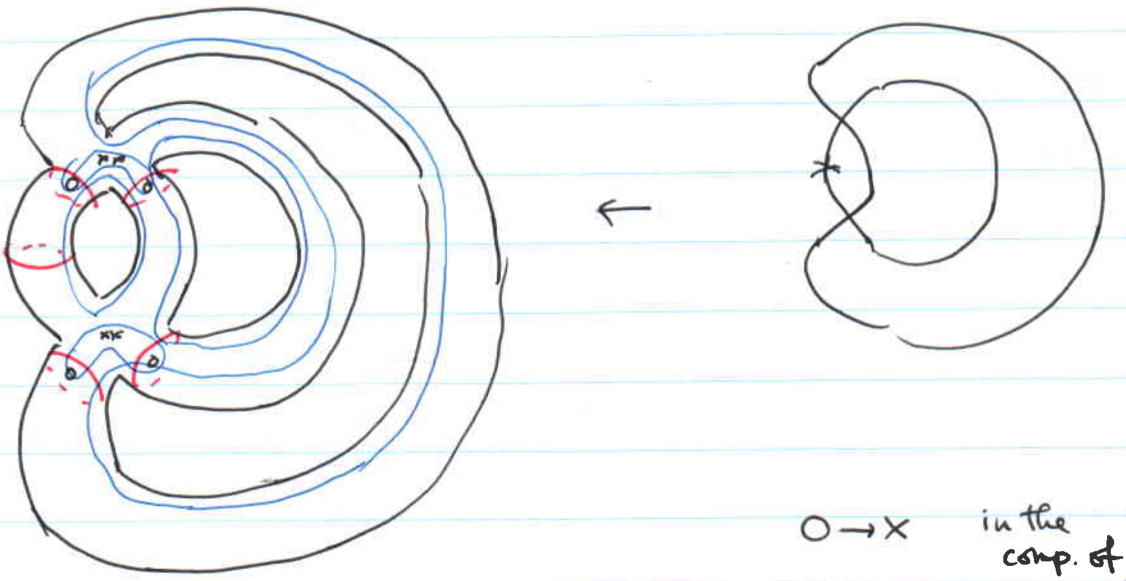
Heegard

$\rightsquigarrow$ each A_i has one X_i or $2X_i$



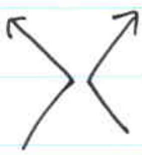
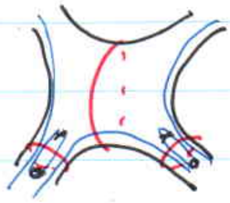






$O \to X$ in the comp. of blue

$X \to O$ in the comp of red

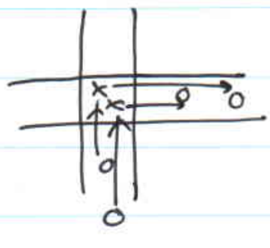
 

$$\mathrm{Sym}^{g+n-1}(\Sigma_g)$$

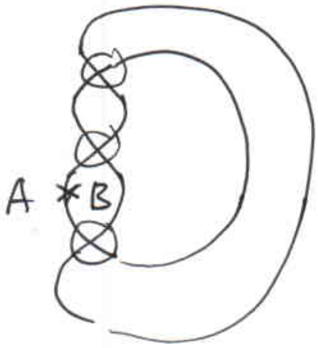
$\mathbb{T}_\alpha \hookrightarrow \mathrm{Sym}^{g+n-1}(\Sigma_g) \hookleftarrow \mathbb{T}_\beta$

$$\partial x = \sum_{y \in \mathbb{T}_\alpha \cap \mathbb{T}_\beta} \sum_{\substack{\varphi \in \pi_2(x,y) \\ X_i \cap \varphi = 0}} \# \frac{\mathcal{M}(\varphi)}{\mathbb{R}} \, U_1^{O_1(\varphi)} \cdots U_n^{O_n(\varphi)} \, y$$

(cf. grid diagram



Kauffmann state

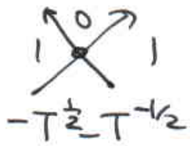


$x \in Q(K) \to R(S^2 - K)$
$- A - D$

x is 1:1

$$\Delta_K = \sum_{x \in Kauf} \prod_{c \in Cr(K)} \text{local contr. of } x \text{ at } c$$

generalization



Then

HFK^- is <u>generated</u> by Kauffman states